

***L1 minimization method for estimating sparsely represented errors
in electric energy system:***

Simulation based investigation of IEEE 300 bus test case with a linearized model

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Outline

- *Motivation*
- *L1 minimization based state estimation for a linear measurement model*
- *Sparsely represented error simulations*
 - *Sparse errors in node voltage angle measurements*
- *Summary*

Motivation

- **State estimation of electric energy systems**
 - *Estimate electrical states of the network from noisy measurements based on given network topology and parameters*
- **State estimation for sparsely represented measurement error**
 - *i.e., arbitrarily large values for a few elements whose sensor may have failed, while zero errors for others*
 - *L0-min. method may be efficient, but L0-min. is an NP-hard problem*
 - *L1-min., polynomial-time, is equivalent to L0-min. when the measurement matrix satisfies a restricted isometric prop. (RIP)*
 - ***Investigation of L1-min. methods using numerical simulation***

State Estimation based on a Linear Measurement Model

- **General state estimation problem***

- **Nonlinear measurement model, $y_j = h_j(\mathbf{x}) + e_j$, e_j : mean 0, variance σ_j^2**
- **Weighted least square (WLS) state estimation**

$$\begin{aligned} \min_{\mathbf{x}} J(\mathbf{x}) &= \sum_{j=1}^m (y_j - h_j(\mathbf{x}))^2 / \sigma_j^2 \\ \text{s.t. } g_i(\mathbf{x}) &= 0; \quad i = 1, \dots, n_g \\ c_i(\mathbf{x}) &\leq 0; \quad i = 1, \dots, n_c \end{aligned}$$

m measurements, n state variables, $n < m$
 $g_i(\cdot)$ and $c_i(\cdot)$ are equality and inequality constraints

- **State estimation for a linear measurement model****

(Linearized Model between node active powers, P , & voltage angles, δ)

$$P_i = -\sum_{j=1, j \neq i}^n |Y_{ij}| (\delta_j - \delta_i) \quad \rightarrow \mathbf{P} = \mathbf{W}\delta$$

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{e}, \quad \mathbf{x} = \begin{bmatrix} \mathbf{P} \\ \delta \end{bmatrix}$$

$$\begin{aligned} \min_{\mathbf{x}} (\mathbf{y} - \mathbf{A}\mathbf{x})^T \mathbf{C}^{-1} (\mathbf{y} - \mathbf{A}\mathbf{x}), \quad \mathbf{C} &= \text{diag}(\sigma_j^2) \\ \text{s.t. } \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & -\mathbf{W} \end{bmatrix} \mathbf{x} &= \mathbf{0} \end{aligned}$$

Estimation of Sparsely Represented Measurement Errors

- **Problem statement**

- **Linear measurements**

$$y = Ax + e, \quad \begin{array}{l} e \text{ is } 2n \times 1 \text{ and sparse, near zero values except } k \text{ elements, } k \ll 2n \\ A \text{ is a } m \times 2n \text{ matrix, } m < 2n \end{array}$$

- **Estimate e from y**

- **L1 minimization method for estimating a sparse error**

- **Sparse errors in node voltage angle measurements and noiseless node active power measurements**

$$y = \begin{bmatrix} P_s \\ \tilde{\delta} \end{bmatrix} = \begin{bmatrix} I_s & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} P \\ \delta \end{bmatrix} + \begin{bmatrix} 0 \\ \Delta\delta \end{bmatrix}, \quad A = \begin{bmatrix} I_s & 0 \\ 0 & I \end{bmatrix}, e = \begin{bmatrix} 0 \\ \Delta\delta \end{bmatrix}$$

I_s is a $m \times n$ matrix, $m < n$,
where P_s is a subset of P , $P_s = I_s P$

$\Delta\delta$ is $n \times 1$ and sparse

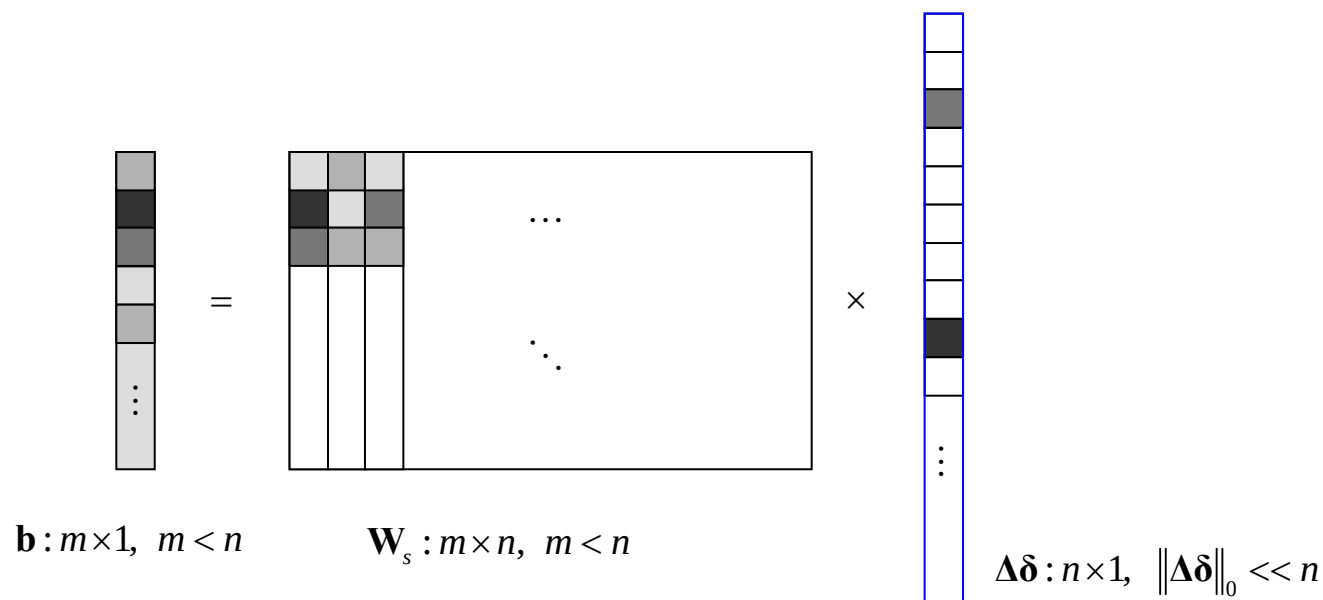
$$\min_{\Delta\delta} \|\Delta\delta\|_1 \quad \text{s.t. } W_s \Delta\delta = b, \quad \begin{array}{l} b = W_s \tilde{\delta} - P_s \\ W_s = I_s W \end{array}$$

* **Least square method :** $\min_{\Delta\delta} \|\Delta\delta\|_2^2 \quad \text{s.t. } W_s \Delta\delta = b$

Estimation of Sparsely Represented Measurement Errors

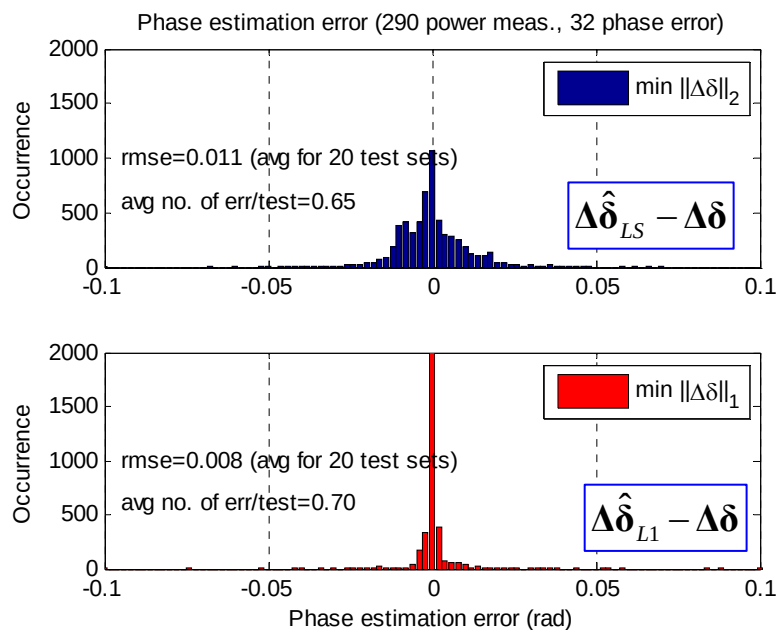
- *Estimating sparsely represented error, $\Delta\delta$, from*

$$\min_{\Delta\delta} \|\Delta\delta\|_1 \quad \text{s.t. } \mathbf{W}_s \Delta\delta = \mathbf{b}$$

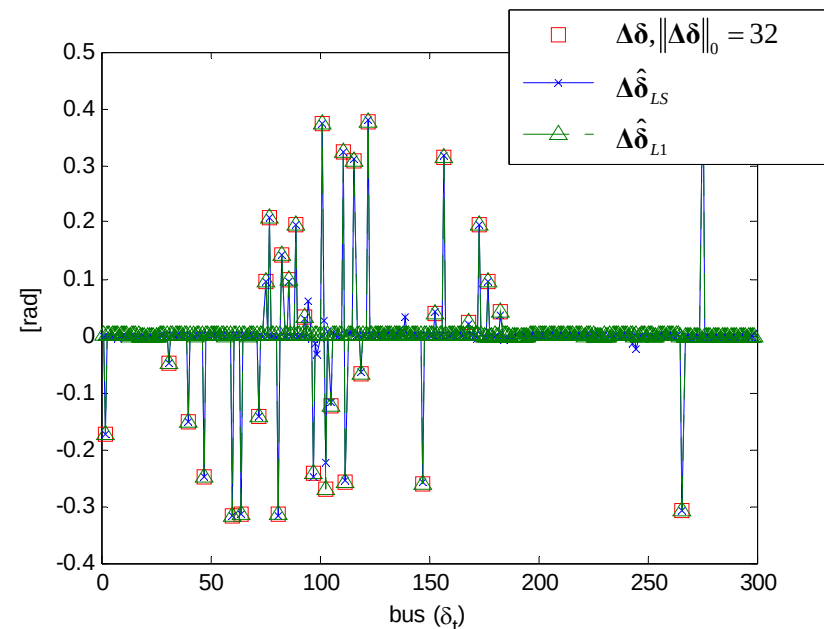


Simulation Results

- **Sparse errors in bus voltage angle meas. (IEEE 300 test case)**
 - **290 power measurements (noiseless)**
 - **32 voltage angle errors, $\Delta\delta_j \sim N(0, 0.25^2 (=var(\delta)))$**



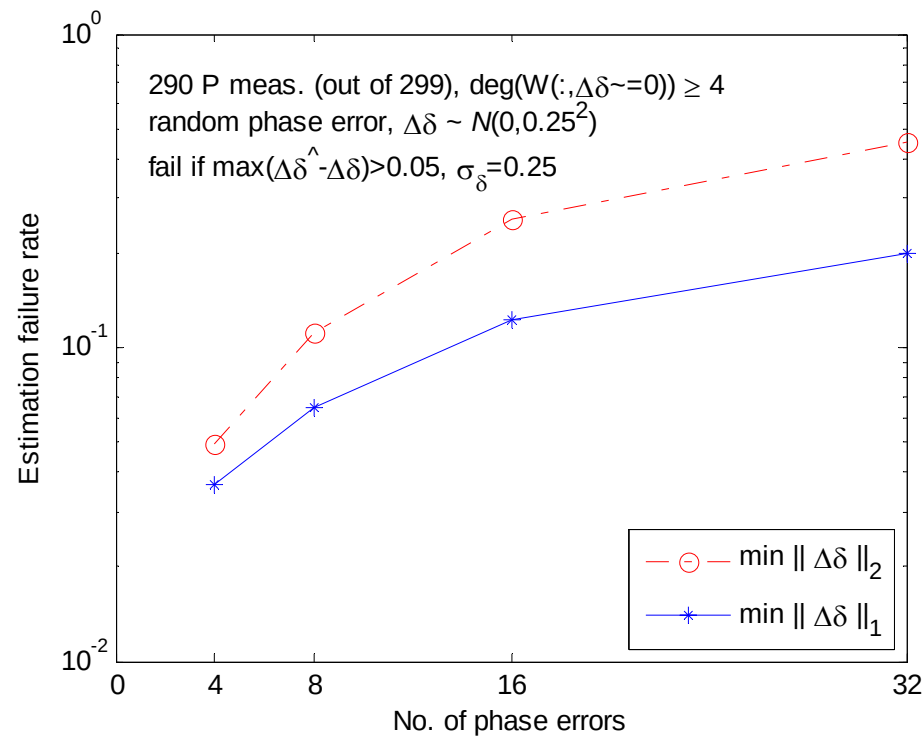
Histogram of voltage angle estimation error
(290 power meas., 32 voltage angle errors, ...)



Injected and estimated voltage angle errors

Simulation Results

- **Sparse errors in bus voltage angle meas. (IEEE 300 test case)**
 - 4, 8, 16 and 32 errors, degrees of voltage angle error columns of $W \geq 4$



Estimation failure rates as a function of the number of voltage angle errors

Summary

- ***Evaluation of IEEE 300 test case with a linearized model***
 - ***Sparse voltage angle errors can be better estimated by L1 minimization method than LS method (290 noiseless power meas. and 4~32 voltage angle errors)***
- ***Further investigations***
 - ***Sparse errors in node voltage angle measurements and Gaussian noise in node active power measurements***
 - ***Subset of node voltage angle measurements with sparsely represented errors***